THICK FUNICULAR: PARTICLE-SPRING SYSTEMS FOR VARIABLE-DEPTH FORM-RESPONDING COMPRESSION-ONLY STRUCTURES

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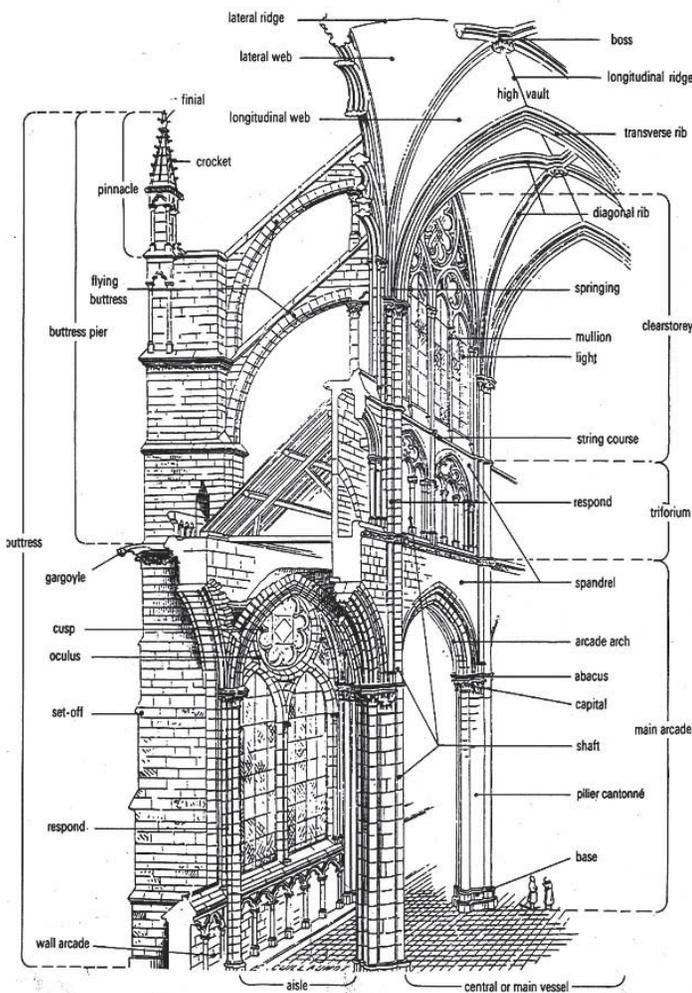Princeton University

INTRODUCTION

Figure 1. Section through Amiens Cathedral.

Thin-shell compression-only structural systems are relatively new to the built environment. Compression-only structures on the other hand are ancient. Thin-shell structures assume a minimal and consistent cross-section. This assumption is driven by material efficiency. The results are forms developed exclusively by structural concerns (typically gravity), hence the term form-found. Architecture has to respond to structural concerns, but it also has to address a variety of other issues — acoustic, formal, program, etc. It is not necessary for form to be driven strictly by structural requirements. For example, Gothic Cathedrals contain the thrust-vector within the variable depth of the stone's cross-section. These Cathedrals are not determined by idealized catenary form, but through a confluence of architectural desires with compression-only principles. With this approach as inspiration, this paper addresses the potentials of compression-only systems to be resolved through a variable-depth in order to obtain a desired form.

Much research has been done in analyzing existing variable-depth structures to determine if a thrust vector falls inside the depth of material (Block et al, 2006). Other methods assume a fixed depth of material in order to generate a design. The method proposed in this paper assumes a desired geometry and allows for a variable thickness to re-direct the thrust vector as a means to produce a viable design that concerns both structure and other formal concerns. If typically one assumes thin, this paper assumes form.

This method is dedicated to addressing architectural concerns with structural results. This paper does not advocate for the reversion to a past architecture. It promotes the insertion of lost knowledge into our current means and methods of making.
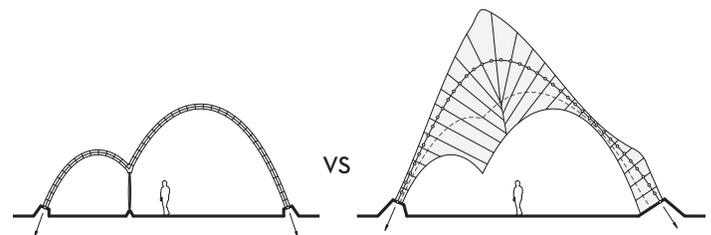

Figure 2. A demonstration of the difference between a thin funicular (bottom) and the proposed thick funicular (top). The thick funicular is able to accommodate the same bottom-geometry without the use of a compression rod.

PARTICLE-SPRING SYSTEMS

Particle-spring systems are based on lumped masses, called particles, which are connected to linear elastic springs. The solver used for this research is part of a particle-spring system implemented by Simon Greenwold (Fry and Reas, 2011). "Each particle in the

system has a position, a velocity, and a variable mass, as well as a summarized vector for all of the forces acting on it" Kilian and Ochsendorf (2005). This Runge-Kutta solver is not necessary to generate a catenary (even load distribution), but it is necessary when evaluating an irregular load case. The method applied in this research will always be an irregular load case because it is assumed the resulting geometry is not an idealized catenary form.

Particle-spring systems have been explored to create virtual form-finding methods such as Kilian's CADenary tool (Kilian and Ochsendorf, 2005).

COMPRESSION-ONLY STRUCTURES

A compression-only structure will stand as long as the thrust vector of the system falls within the middle third of its cross section. It is possible for a system to stand if the thrust vector lands between the middle third and the outer surface; however, it is likely to develop a hinge. For the purposes of this paper, we will maintain a thrust vector inside the middle third of the structure, assuring a result with zero tension.

It is not always predictable that a structure will fail, though it is possible to know if it will stand. A paper (Heyman, 1966) introduced the safe theorem for masonry structures. This theorem states that a compression-only structure can stand so long as one network of compression forces can be found in equilibrium within the section of the structure. This solution is a possible lower-bound solution. When evaluating existing structures, it is not always pos-

sible to understand where exactly this force network is (Block and Ochsendorf, 2008). The method applied in this paper can calculate and assure a thrust vector falls within the thickness of material; however, it cannot guarantee one will not. Because of this uncertainty, a number of assumed failures did not fail.

For further reading on lower-bound analysis of unreinforced masonry structures, see (Heyman, 1982) and (Huera, 2001, 2004).

FORM RESPONDING

Form-finding analog models by such researchers as Otto and Gaudi, or even the virtual versions like Kilian's CADenary (Kilian and Ochsendorf, 2005) have proved it is difficult to control and predict the results of the final found-form. Moreover, if that form does not correspond with a force that is external to the form-finding model, it is difficult to resolve the two into a solution. This paper proposes form-responding as approach. Form-responding takes a desired form as input and produces a variable-depth to allow for interaction between these external forces and the solver-based model.

METHODOLOGY

The method applied in this paper communicates a dynamic relaxation of the particle-spring system in response to a desired base-geometry. This system uses a solver-based approach to inform each particle in the system with the data it requires to compute a solution. The model can be described by the following categories — Base Geometry, Particle-Spring System, Vertical Distance, Expan-
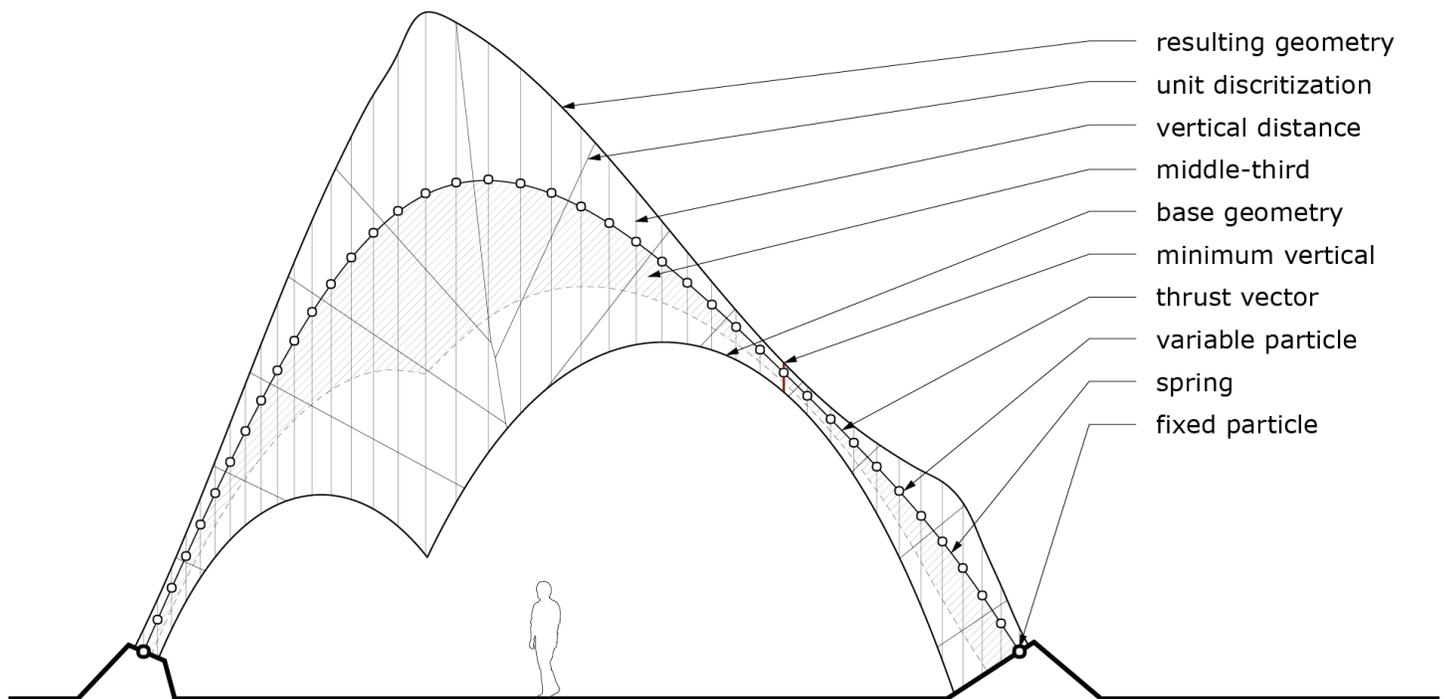

resulting geometry
unit discritization
vertical distance
middle-third
base geometry
minimum vertical
thrust vector
variable particle
spring
fixed particle

Figure 3. Key of the various components in the system.

sion/Contraction, Unit Discritization.

Base Geometry

This paper assumes the base geometry as fixed. The assumption is that this geometry has been pre-determined by a force external to the model — acoustics, formal, building-code, etc. Future research could allow for a more fluid and reciprocal relationship between the structural requirements and these other formal drivers. Not all geometries are currently possible. For instance, vertical undercut is not yet a possible geometry to solve with this method because the vertical intersection rule (see 5.3) does not calculate any mass below the first intersection. Another constraint is resolution. The more volatile this geometry is, the more likely it is for a particle to skip over a nuance in the geometry. Increasing the number of particles as a way to increase resolution can solve this. Smooth curves with a minimal change in radius tend to work best as they compensate for the discrepancies between two particles easily.

A base geometry is required as input to the system. This base geometry is the datum with which the particle system computes it-self against. Experiments have been conducted with both 2D curves and 3D surfaces.

Particle-Spring System

In Processing, a simple five-node b-spline serves as the base geometry. The middle node of this b-spline is interactive. By hovering with the curser, the clicking and dragging, it is possible for the user to manipulate the curve dynamically to interact with the system, and better understand how that geometry informs the particle-spring system. While this was a helpful demonstrative device, the dynamism is confusing. This paper proposes to create a hard geometry in which the particle system responds against. The interactivity is intended for demonstration, not as method.

Rhinoceros and Grasshopper are also used to communicate with Processing via UDP in order to respond to polylines, circles, tilted ellipses, and other geometries.

The number of particles, the length of the springs, and the continual resulting forces on each particle inform the particle-spring system. While this organization is consistent, the system can be re-configured to produce a variety of potential solutions irrespective of the base-geometry. This model can produce 2D arches (Clifford, 2011) but it can also produce aggregated arch vaults, networked mesh vaults, as well as composite solutions.
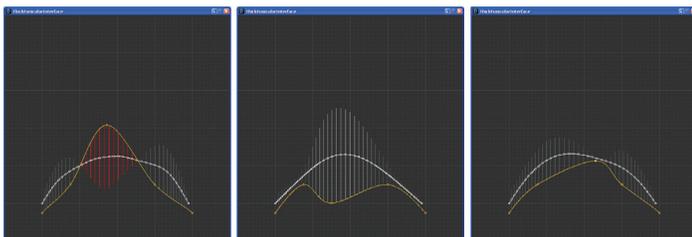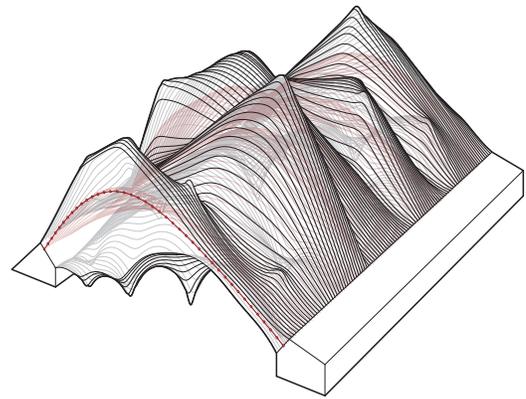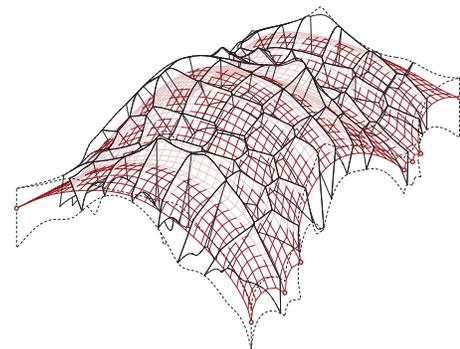

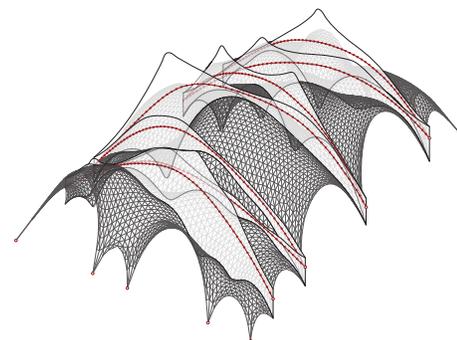Figure 4. Screenshots of the 2D processing interface.

2.5D Aggregated

Figure 5. Drawing of an Aggregated Arch Vault.

3D Network

Figure 6. Drawing of a Networked Mesh Vault.

Composite

Figure 7. Drawing of a Composite Vault.

The aggregated arch vaults are simply the aggregation of a series of independent arches in response to a single base-geometry. These arches are not dependant on the others to stand. This approach has been tested in both parallel and perpendicular configurations. (see 6) While the perpendicular appears to be similar to the networked mesh, these arches slip past each other and are not connected as fixed nodes in the calculation.

The networked mesh vault is a truly three-dimensional problem. While unwieldy and difficult to predict, it is possible to result in a solution with less material depth than the aggregated arch. The advantage the 2D system has is it is calculating a minimal cross section (see 5.4) across the particles of a single arch, where-as the mesh has to expand and contract the entire system until that minimum dimension is met.

It is also possible to combine and reconfigure any of these organizations into a new solution method. The ribbed vault is an example of such a composite. The ribs are performing as thick-funiculars, and the surfaces between are evenly distributed mesh networks.

Vertical Distance

The spire of a Gothic Cathedral has a deeper cross section, and therefore contains more mass, re-directing the thrust vector down into its column. This model is similar in that the depth of material is highly variable producing a non-uniform load on the system (not a catenary). In order to simulate this, the model calculates a vertical intersection between particle and the base-geometry. Figure 8 demonstrates the distance between the particle and this intersection becomes the new vertical thrust vector for that particle. This calculation is continually calculated as the model finds equilibrium.

When analyzing masonry arches, it is common practice to use static block analysis to break down an arch into a few polygons. The area of each polygon determines the vertical thrust vector (Block et al, 2006; Block and Ochsendorf , 2007). This paper relies on the high resolution of the particle-spring system; informing each particle with its vertical distance, not a polygonal area.

Expansion/ Contraction

In order to determine a solution where the thrust-vector falls above the base-geometry, the length of each spring needs to expand evenly throughout the system. It is possible to over expand the lengths, producing a viable solution with an extremely large mass, so a balance needs to be reached in order to find an efficient solution. The designer prescribes a minimal cross section dimension that can be informed by the compression capacity of the material. The system continually searches for the shortest vertical dimension. If that dimension is shorter or longer than the required minimum, the entire system expands or contracts. (Figure 2)
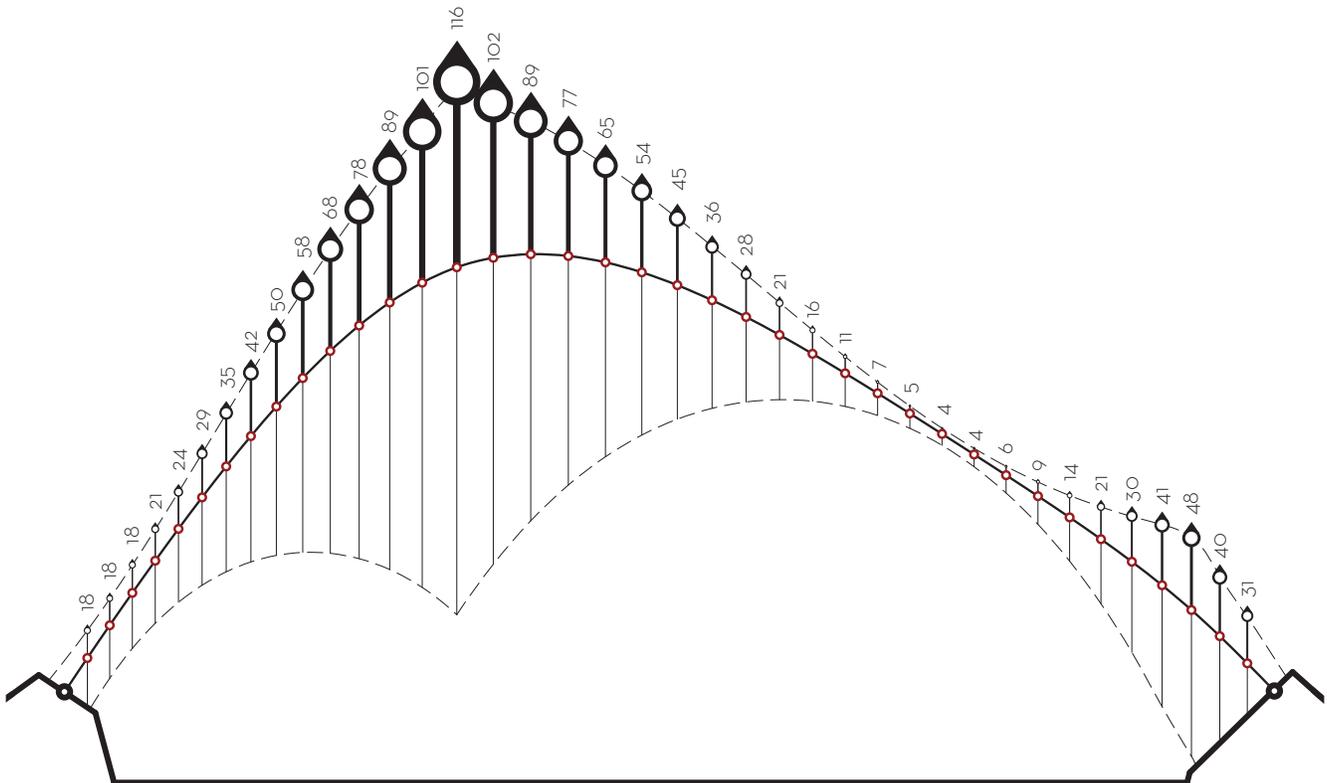

Figure 8. Diagram of the Vertical Distance calculation. The vertical distance between a particle and the base geometry equals the vertical thrust on that particle. d=F

Unit Discretization

Once a desired solution is obtained, it is necessary to break the system down into constructible blocks. In order to resolve the equilibrium of shear forces (and not have to rely on friction) the break lines are created perpendicular to the thrust-vector. These break lines have been evenly spaced, but the number of breaks, and the regularity of these breaks do not alter the solution. When working with 3D systems, a thrust-network surface is created in order to generate break geometries normal to this surface. (Figure 9) As long as these breaks are perpendicular (or normal) to the thrust vector, the system can assure a zero shear solution, meaning blocks can be placed without relying on glue or tension in the system.
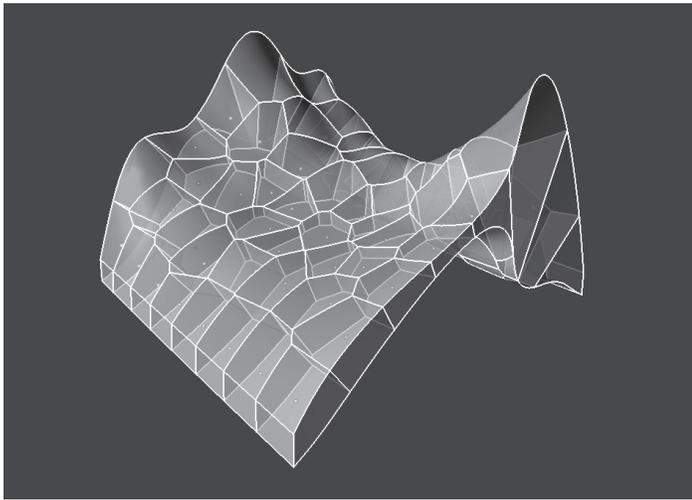
Figure 9. Three-dimensional unit discretization based on the thrust network surface normals.

PROTOTYPING

As this is a scale-less calculation, it has been possible to rapidly prototype each of the various configurations of the system at a small scale. These prototypes can be constructed from a variety of materials as long as they have a consistent density. The material
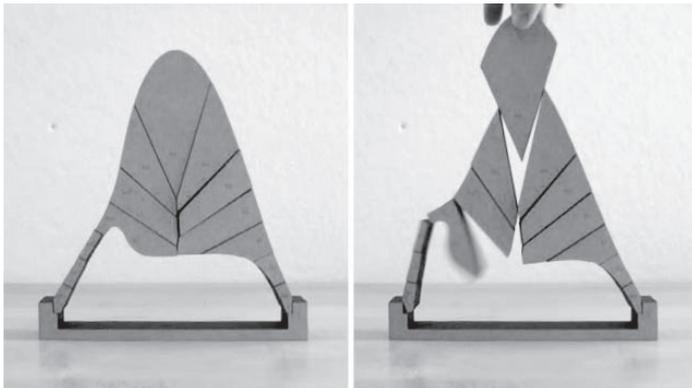
must also be able to accept compression loads without deflection. Prototypes have been constructed from aggregated sheet materials (Figures 10-12) and solid rapid prototyping for the three-dimensional calculations.

Each prototype is erected without the aid of glue or fasteners. This verifies the system functions as a compression-only system. In order to demonstrate this, units are removed and the hinge is detected by recording the failure.
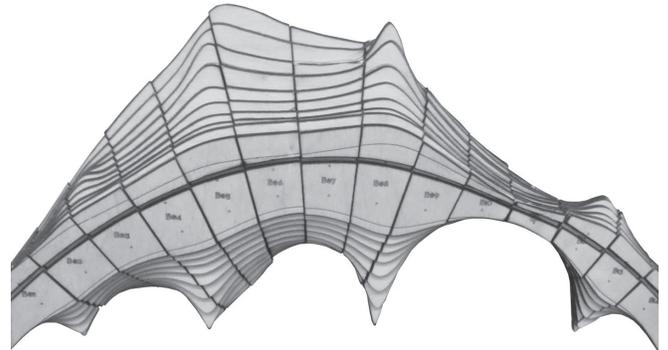

Figure 11. Prototype of an Aggregated Arch Vault generated from a series of parallel particle-spring arches over a continuous surface.
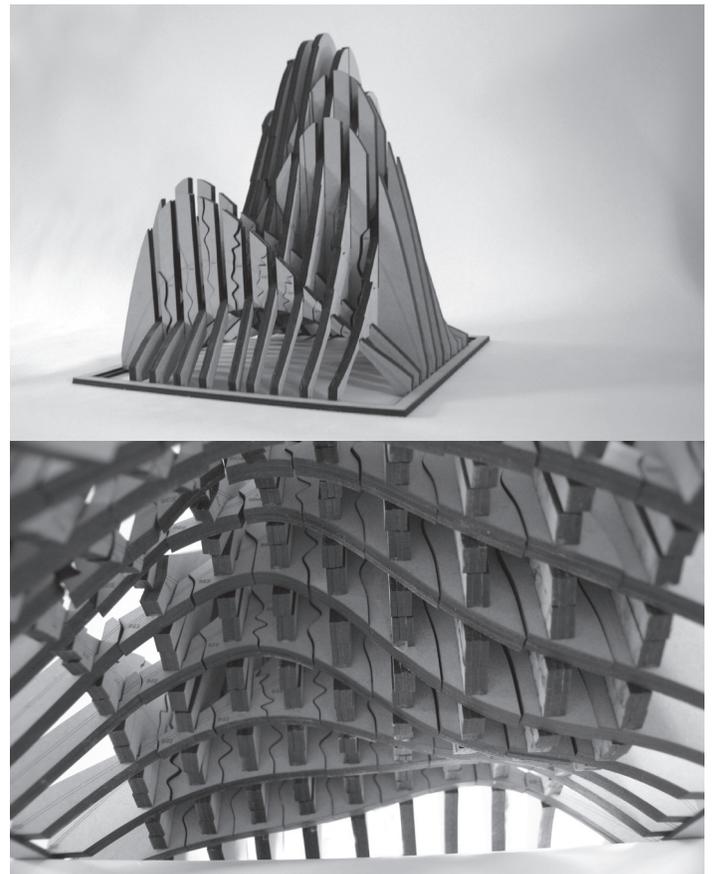

Figure 12. Prototype of an Aggregated Arch Vault generated from a series of perpendicular particle-spring arches over a continuous surface.

Figure 10. Prototype of a two-dimensional Arch.

Material

It is possible to prototype these solutions from a variety of materials. Because the system is informing each particle with a linear vertical dimension, the only requirements are that the materials be of consistent mass in order to produce a relative result. It is also a requirement the material be able to accept compression loads without significant deflection. Prototypes were constructed from MDF sheet and plastic 3D prints.

Erection Process

Early prototypes were constructed with formwork dropped vertically after the units were placed. A number of problems arose with this drop mechanism. Figure 13 captures the dropping of one side faster than the other, pushing the shoring into the blocks on one side, resulting in a failure. Later experiments negated the need for this formwork by incorporating a raceway for a string to post-tension and erect the system. Once erect the string is removed to allow the thick funicular to maintain a compression-only solution. (Figure 14)
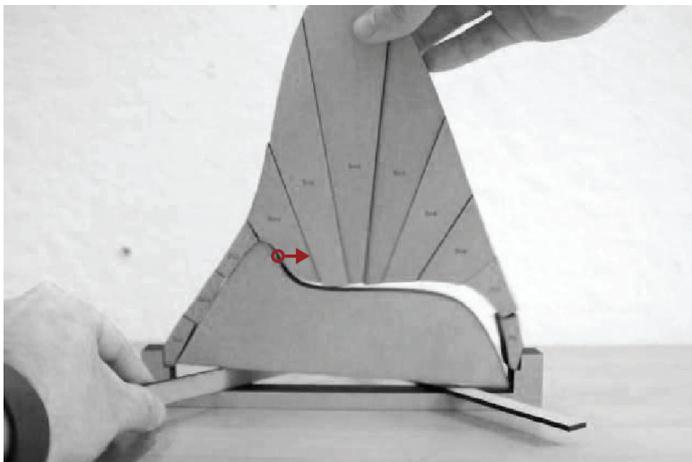
Figure 13. A collapse caused as the drop mechanism did not fall straight down.

Figure 14. Post tension erection prototype. This method negates the need for formwork.

ANALYSIS

Not all of the prototypes would stand. Various theories were developed to explain why. These theories ranged from human error, to unit buckling in the z-direction. Videotaping these failures was helpful to play back in slow motion to verify the theories.

Additionally, some arches created specifically to demonstrate failure, did not fail. As mentioned previously, (see 3) as long as there is a lower bound solution, the arch will stand. This method currently guarantees a trust vector can fall within the thickness of material, but cannot guarantee one does not.

In order to verify the variable depth was contributing to ensuring the thrust vector remained inside the middle-third of the cross-section, it was necessary to produce a calculated failure. For this prototype, a pair of isolated loads informed a few particles on each side of the arch. These particles were overwritten in the code to understand they were to be two or sometimes four times more impacted by the vertical thrust. (Figure 15) When the mass was removed, the arch exposed a hinge point where the new thrust vector fell outside of the arch cross-section, demonstrating the variable mass was required to make the system stand.
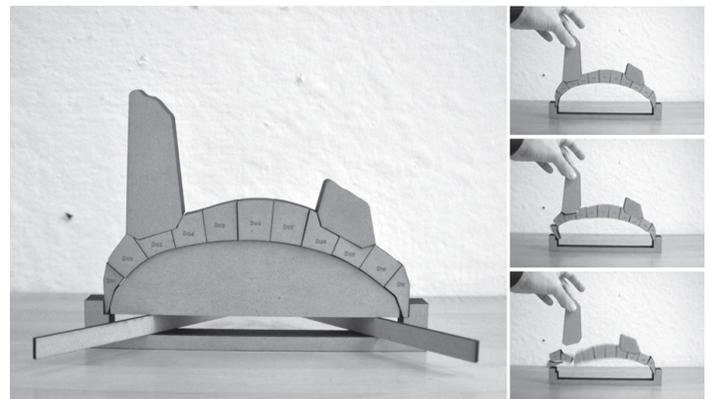
Figure 15. Prototype of an isolated external load case. By removing this load, the photo captures the hinges generated as the thrust vector leaves the thickness of the middle-third.

CONCLUSION

This paper demonstrates the potential to use a particle-spring system to generate a constructible compression-only structure that is not simply dedicated to material efficiently, but to the capacity to respond to other concerns architecture is required to address. While these concerns have not been inserted into this research, the placeholder (base-geometry) for these concerns is a venue for further exploration.

The system currently produces a single solution, though there are an infinite number of possible results. It is possible to expand and contract the system to accommodate varied minimum dimensions. It is also possible to add isolated load cases. Each variable is capable of producing a new solution. Future research could produce an array of solutions and select the one with the least mass or material as the most efficient solution.

The variable depth of the solutions also creates a number of problems with fabrication that need to be resolved. At the relatively

small scale of these prototypes, aggregating sheet material and 3d rapid prototyping is currently a placeholder for this problem of fabricating with volumetric materials. Future research will inform the model with the means and methods of making at full scale with materials such as AAC (aerated autoclaved concrete).

The results of this model are often extremely thick. While variable thickness is strongly advocated here, a question has arisen about the reality of this extreme depth change. Future research also resides in the potential to calculate a variable volume as opposed to thickness in order to accommodate a less exaggerated solution.

ACKNOWLEDGEMENTS

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